

RIPE60 – Rate Independent Processes and Evolution Workshop on the occasion of Pavel Krejčí 60th birthday

24 – 26 June 2014, Prague

Institute of Mathematics of the Academy of Sciences of the Czech Republic

Titles and abstracts of lectures

Martin Brokate: *Differential sensitivity of systems with scalar hysteresis*

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Abstract: We discuss weak differentiability properties of scalar hysteresis operators as well as some consequences for dynamical systems with hysteresis.

Jan Chleboun: *A note on optimal node and polynomial degree distribution in one-dimensional hp-FEM*

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Abstract: The goal is to construct an optimal higher-order finite element mesh under a constraint on the total number of degrees of freedom (DOF). The motivation is to obtain a truly optimal higher-order finite element mesh that can be used to compare the quality of automatic adaptive algorithms. For a 1D model problem, the approximation error in a global norm is minimized. Optimization variables comprise the number of elements, the positions of nodes, and the polynomial degrees of elements. Although optimal settings are found, it turns out that, due to computational complexity, the optimization goal is achievable only for a small number of DOF.

Pierluigi Colli: *Phase field systems of viscous Cahn-Hilliard type*

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Abstract: A nonstandard phase field model of Cahn-Hilliard type is considered. The model describes two-species phase segregation and consists of a system of two highly nonlinearly coupled PDEs. The system of differential equations, complemented by initial and boundary conditions, will be discussed and some results that have been obtained for this class of problems will be reviewed.

Daniele Davino: *Identification of a new class of operators aimed to the modelling of smart materials*

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Abstract: Smart materials, as magnetostrictives or piezoelectrics, are very interesting for many engineering fields in sensoric, actuation and energy harvesting tasks. The talk will present the use of a general class of hysteresis operators to describe the behaviour of a piezoelectric stack actuator. The model relates both the electric and mechanical characteristics taking into account a thermodynamic compatibility. In the talk, after a short introduction, the implementation and its agreement with experiments will be presented.

Pavel Drábek: *Travelling waves in quasilinear FKPP equation*

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Abstract: We prove the existence and uniqueness of a family of travelling waves in a degenerate (or singular) quasilinear parabolic problem that may be regarded as a generalization of the semilinear Fisher-Kolmogorov-Petrovski-Piscounov equation for the advance of advantageous genes in biology. Depending on the relation between the nonlinear diffusion and the nonsmooth reaction function, which we quantify precisely, we investigate the shape and asymptotic properties of travelling waves. Our method is based on comparison results for semilinear ODEs.

Michela Eleuteri: 2004–2014 Story of a ten-years collaboration (with hysteresis)

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Abstract: The main results obtained in cooperation with Pavel Krejčí will be presented. The topic concerns partial differential equations with hysteresis applied to models of soil hydrology, magnetohydrodynamics and material fatigue.

Jan Franců: Reliable solution of problems with hysteresis operators

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Abstract: The contribution deals with boundary value problems for scalar wave and diffusion differential equations with Prandtl-Ishlinskii operator. Since parameters of the operator are known in some extent only, the method of worst-scenario by Babuška and Hlaváček is applied. It proves existence of the reliable solution. The corresponding homogenization problem is solved as well.

Sergio Frigeri: Cahn-Hilliard-Navier-Stokes systems with nonlocal interactions and optimal control

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Abstract: In the talk some results on well-posedness and long-time behaviour for nonlocal Cahn-Hilliard-Navier-Stokes systems with different assumptions on mobility, viscosity and double well potential will be first recalled. Then, some new results concerning the optimal control for the system in 2D will be presented.

Gianni Gilardi: On a Cahn-Hilliard type phase field system related to tumor growth

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Abstract: A model related to tumor growth has been recently presented by D. Hilhorst. The corresponding boundary value problem couples a nonlinear parabolic equation and a Cahn-Hilliard type equation and it depends on two small diffusion parameters. In this talk, some related results obtained in a joint paper with P. Colli and D. Hilhorst will be presented. Namely, well posedness and long time behaviour in the case of positive diffusion coefficients will be discussed. Moreover, the asymptotic analysis as the diffusion parameters tends to zero and the limiting problem are studied. In the last direction, also the first results of a joint work in progress with P. Colli, E. Rocca and J. Sprekels are presented.

Maurizio Grasselli: Nonlocal Cahn-Hilliard-Navier-Stokes systems: the 2D case

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Abstract: We consider a diffuse interface model for incompressible isothermal mixtures of two immiscible fluids with matched densities. This model consists of the Navier-Stokes system coupled with a convective nonlocal Cahn-Hilliard equation. We present the state-of-the-art in the two-dimensional case. The resulting global picture is due to strong local and nonlocal interactions with P. Colli, C.G. Gal, S. Frigeri, E. Rocca and, of course, P. Krejčí.

Dietmar Hömberg: Models of induction hardening – an FK limited approach

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Barbara Kaltenbacher: Well-posedness for a thermodynamically consistent model of ferroelectricity and ferroelasticity

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Abstract: The use of hysteresis potentials allows designing thermodynamically consistent models for smart materials as has been shown in a paper by Daniele Davino, Pavel Krejčí, and Ciro Visone (Smart Materials and Structures, 2013) in the context of magnetostriction. This model can directly be carried over to piezoelectricity, where hysteresis is due to ferroelectricity and ferroelasticity. In this talk we

will dwell on a well-posedness proof for the system of PDEs resulting from the mentioned material law, combined with the relevant balance equations (namely Newton's law on the mechanical side and Gauss's law on the electric side). The presented results are based on recent joint work with Pavel Krejčí.

Olaf Klein: *Classification of hysteresis operators for vector-valued inputs by using their representation as functions on strings*

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Abstract: In [1], Brokate and Sprekels have proved that hysteresis operators acting on scalar-valued continuous input functions being piecewise monotone can be represented by functions on alternating strings. In [2]–[3], a corresponding representation result is derived for hysteresis operators acting on vector valued continuous input functions, being piecewise monotaffine, i.e. being piecewise the composition of two functions such that the output of a monotone increasing function is used as input for an affine function. These operators can be represented by functions acting on the set of convexity triple free strings. This representation result allows formulating a vector version of the forgetting according to Madelung deletion considered in [1]. Also the congruence property of vector minor loops considered in [4] can be rewritten as a condition for the representation. These results will be applied to investigate the properties of some hysteresis operators including the generalized vectorial relay and the continuous Mróz hardening rule.

- [1] M. Brokate and J. Sprekels. Hysteresis and phase transitions. Springer, New York, 1996.
- [2] O. Klein. Representation of hysteresis operators acting on vector-valued monotaffine functions. Adv. Math. Sci. Appl., 22: 471–500, 2012.
- [3] O. Klein. Darstellung von Hysterese-Operatoren mit stückweise monotaffinen Input-Funktionen durch Funktionen auf Strings. Submitted habilitation thesis, Humboldt-Universität zu Berlin, 2013.
- [4] I. D. Mayergoyz. Mathematical Models of Hysteresis and their Applications. Elsevier, second edition edition, 2003.

Jana Kopfová: *Hysteresis in different modelling situations - my math journey with Pavel*

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Abstract: I will talk about my math journey with Pavel – things and approaches I have learned from him and about the mathematical results of our cooperation. I will start with Pavel's formulation of the Preisach operator and show how it can be used to get a homogenization result for a parabolic equation with Preisach nonlinearity. I will also present our joint results on the magnetohydrodynamic models with Preisach hysteresis nonlinearities. A new temperature dependent Preisach model was another direction of our research, and I will briefly introduce the main idea. At the end I will talk about our results on the fatigue modelling in oscillating elastoplastic materials, and in combination with a contact boundary condition.

Klaus Kuhnen: *Rate-independent dissipation in power electronic circuits*

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Abstract: The increasing demand on cost and volume reduced power electronics in automotive applications leads to increasing switching frequencies for the semiconductors and an increasing miniaturisation of the passive energy storage components. The consequence of this evolution is a higher stress of the magnetic and dielectric materials in the inductive or capacitive components during operation and thus a higher dissipation per volume. This is the reason why the thermal domain becomes more and more a bottleneck for a further miniaturisation. Therefore, a further optimization of power electronic circuits with respect to lower costs, smaller volumes and higher power densities requires precise models for the calculation of dissipation effects which are the main heat sources in the circuit. One important source of dissipation between many others is the hysteretic loss in ferromagnetic or ferroelectric materials. This contribution discusses the use of

different scalar hysteresis operators as models for the material characteristic together with the associated hysteretic potential and rate-independent dissipation operator as a model for the energy storage and source for the thermal domain.

Philippe Laurençot: *A free boundary problem modelling a microelectromechanical system (MEMS)*

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Abstract: A simplified MEMS device is considered, which is made of a deformable membrane covered by a thin conducting material suspended above a rigid conducting plate. Applying a potential difference yields a deformation of the membrane and the behaviour of the device is described by the evolution of the membrane deformation and the electrostatic potential in the device. The resulting mathematical model couples a parabolic or hyperbolic equation for the deformation with an elliptic equation on a time-dependent domain for the potential. Existence and non-existence results will be presented. Joint work with J. Escher and Ch. Walker (Hannover).

Matthias Liero: *On dissipation distances for reaction-diffusion equations – the Hellinger-Kantorovich distance*, joint work with Alexander Mielke (Berlin) and Giuseppe Savaré (Pavia)

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Abstract: Since the late nineties it has been known that certain diffusion processes can be interpreted as gradient flows with respect to the relative entropy and the Kantorovich-Wasserstein distance. Up to now, a rich theory for this geometric view on (scalar) diffusion processes within the framework of optimal transport was developed (see [AGS05]). More recently, Mielke showed in [Mie11] that a wide class of reaction-diffusion systems can be formulated in a natural way via gradient structures for the relative entropy. The metric gradient is determined via a state-dependent Onsager operator containing a diffusion part of Wasserstein type and an additional reaction term. Given such an Onsager operator we can introduce a dissipation distance in the sense of Benamou-Brenier by infimizing the total dissipation over all connecting curves (see [BB00]). The question of attainment of this infimum, which is the same as the existence of geodesic curves, is an open question in most cases. In this talk we present first results for the most simple scalar reaction-diffusion equation $\dot{u} = \Delta u - f(u)$. The associated Onsager operator gives rise to the Hellinger-Kantorovich distance, which is the inf-convolution of the Kantorovich-Wasserstein and the Hellinger distance. It is described as a function of any two nonnegative measures in such a way that the competition between transport and reaction is clearly displayed. We derive a characterization of the distance in terms of a minimization of a coupling measure and a cost function describing the transport part. The crucial point is to identify the amount of transport versus the amount of reaction. Finally, we present some examples of Hellinger-Kantorovich geodesic curves between measures that highlight the competing effects.

[AGS05] L. Ambrosio, N. Gigli, and G. Savaré. Gradient flows in metric spaces and in the space of probability measures. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2005.

[BB00] J.-D. Benamou and Y. Brenier. A computational fluid mechanics solution to the Monge-Kantorovich mass transfer problem. *Numer. Math.*, 84(3), 375-393, 2000.

[Mie11] A. Mielke. A gradient structure for reaction-diffusion systems and for energy-drift-diffusion systems. *Nonlinearity*, 24, 1329 - 1346, 2011.

Alain Miranville: *Recent results on the Cahn-Hilliard equation and some of its variants*

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Abstract: Our aim in this talk is to discuss recent results on the Cahn-Hilliard equation and some of its variants. These equations have applications in phase separation, biology, image inpainting.

Adrien Petrov: *Solvability for elasto-plastic contact problems with heat transfer*

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Abstract: In this talk, a dynamical model for the motion of a visco-elasto-plastic material in contact with an elasto-plastic obstacle and undergoing thermal expansion is introduced. The problem for the unknown displacement and temperature is written in accordance with the formalism of hysteresis operators as solution operators of the underlying variational inequalities. Under appropriate regularity assumptions on the data, the existence and uniqueness results for this thermodynamically consistent problem are established. More precisely, a space discretization is introduced and some a priori estimates are obtained by using both the classical energy estimate and more specific techniques like the Dafermos estimate [Daf82] as well as the Sobolev interpolation inequalities (see [BIN78, KrP11]) leading to the existence result. The uniqueness result follows from the Lipschitz continuity of the nonlinearities. This is a joint work with Pavel Krejčí.

[BIN78] O. V. Besov, V. P. Il'in and S. M. Nikol'skiĭ. Integral Representations of Functions and Imbedding Theorems. Scripta Series in Mathematics, Halsted Press (John Wiley & Sons), New York-Toronto, Ont.-London, 1978 (Vol. I), 1979 (Vol. II). Russian version Nauka, Moscow, 1975.

[Daf82] C. M. Dafermos. Global smooth solutions to the initial-boundary value problem for the equations of one-dimensional thermoviscoelasticity. SIAM J. Math. Anal. 13 (1982), 397–408.

[KrP11] P. Krejčí and L. Panizzi. Regularity and uniqueness in quasi-linear parabolic systems. Appl. Math. 56, 341–370, 2011.

Hana Petzeltová: *Mathematical analysis of variable density flows in porous media*

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Abstract: We consider a simple model describing the motion of a two-component mixture through a porous medium. We discuss well-posedness of the associated initial-boundary value problem, in particular, with respect to the choice of boundary conditions. The existence of global-in-time solutions is proved in the ideal case when the fluid occupies the whole physical space. Finally, similar results are obtained also for the boundary value problems in the simplified 1-D geometry.

Dmitrii Rachinskii: *Discontinuous Prandtl-Ishlinskii operators and their network connections*

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Abstract: Prandtl-Ishlinskii (PI) operators are used to model constitutive laws of elastoplastic materials, friction, fatigue, and, more recently, complex constitutive laws of smart materials. The term "Prandtl-Ishlinskii operator" was coined by Pavel Krejčí. He also showed that the class of monotone PI operators is closed with respect to the composition operation. This result included a simple formula for inversion of the PI operator and has found various applications to compensation based control of sensors and actuators employing smart materials for energy conversion. In other words, a cascade connection of PI operators produces a PI operator. On the other hand, just by definition, any linear combination (parallel connection) of PI operators also belongs to the class of PI operators. In this paper, we consider network connections of interacting PI operators. This setting naturally leads us to a discontinuous extension of the PI operator. We show that, under natural conditions, the network of PI operators is equivalent to one effective (discontinuous) PI operator. However, if these conditions are violated, the networked system can lose the return point memory property and would produce more complex hysteresis loops than any PI operator. These ideas will be illustrated by prototype models of mechanical and financial systems.

Vincenzo Recupero: *Polyhedra and variational evolution limits*

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Abstract: BV solutions of some classes of evolution variational inequalities can be obtained as limit of simpler classes of solutions in suitable topologies. For the play operator two notions of BV solutions

naturally arise if we consider the topology of uniform convergence and the strict topology. We compare these two notions and we show that they coincide if and only if the characteristic set is a non-obtuse polyhedron. This is a joint work with Pavel Krejčí.

Tomáš Roubíček: *Various solution concepts in rate-independent evolution systems*

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Abstract: Evolution of mechanical systems is often governed by nonconvex stored energies together with dissipation potentials. If the outer loading is much slower than the time-scale of internal dissipative processes, these systems can approximately be considered as rate independent and the dissipated energy potential as positively homogenous of degree-1. Typical examples are damage, plasticity, phase transformations, or fracture in the bulk, or delamination or friction in adhesive contacts. The usual global-minimum concept for incremental problems preserves energy in the limit but may be computationally difficult and often less physical than some force-driven locally-minimal solutions. Various concepts of solutions and various time-discretisations will be discussed, together with the role of the maximum-dissipation principle. Abstract considerations will be illustrated on a delamination problem (= an adhesive contact problem), together with some of its variants as a brittle contact or a mixity-mode sensitive delamination (illustrated by numerical experiments performed by C.G. Panagiotopoulos and R. Vodička).

Giulio Schimperna: *On a general class of doubly nonlinear equations*

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Abstract: In this talk, we will present an existence theorem for weak solutions to a class of doubly nonlinear parabolic evolution equations of the form $A(u_t) + B(u) = f$. Our main application refers to the case when B is the so-called $m(x)$ -Laplacian, while $A(u_t) = |u_t|^{p(x)-2} u_t$; namely, we consider growth exponents m and p depending on the variable x . The proof of our result is based on the derivation of suitable approximation and integration by parts formulas in variable-exponent Lebesgue spaces. We will also discuss the possibility to consider even more general classes of operators A including dependence also on the time variable t . These results have been obtained in collaboration with Goro Akagi (University of Kobe).

Jürgen Sprekels: *Experiencing the friendship of a great personality: almost twenty years of cooperation with Pavel Krejčí*

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Ivan Straškraba: *Two Phase Flow in Hydraulics*

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Abstract: A system describing fluid flow in a simple hydraulic system consisting of a pipe with generator on one side and a valve on the other side of the pipe is studied. Originally strongly nonlinear system is linearized around a fixed steady state solution. Note that steady state solutions have already been analysed in our previous investigation (to appear in Applications of Mathematics). Arising system of partial differential equations is of mixed type, more or less hyperbolic-parabolic. Combined techniques are used to find as explicit result as possible.

Ciro Visone: *The mathematics of hysteresis in the progress of analysis and design of devices exploiting non-reversible processes*

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Abstract: Until some decades ago, the required modeling effort in the analysis of devices was to consider the employed materials as linear or, at most, as nonlinear and *memoryless*. However, when a higher accuracy was required, the modeling effort should become more and more demanding. For

these reasons, we assisted in the late '80s to an increasing interest to models of systems and processes with *rate-independent* memory. Models proposed at the beginning of 20th century, such as Preisach and Prandtl-Ishilinskii, stated in a formal fashion by M. A. Krasnoselskii in the '70s were "rediscovered" and widely applied in the analysis and design of technological devices. In these frame we can locate the effort of several mathematicians who developed and clarified several notions, such as the idea of *memory rules*, *state* and *inverse* that assumed an outstanding importance in the engineering community. The discussion would so provide a survey of some of these concepts, from the technologist viewpoint, that resulted irreplaceable in several fields of engineering and allowed to improve the design of electromagnetic devices or the control of systems employing smart materials.